AUTOMATIC SOLUTION OF TRANSPORT PROBLEMS WITH EXCEL

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Abstract: Transport problem is a basic problem connected with the transportation of products from several distributors to several customers. There are paths between each pair distributor-customer. Let each distributor has some determined preliminary known quantity of particular type of product and all the quantity of this product has to be supplied to each customer according to its necessity. The solution of this problem is to obtain the optimal transportation and allocation the products. In this paper is suggested an automatic solution of all different types of transport problems by using Excel.

The advantages of this solution consist of significant decrease of the calculation time and easily and good visualization of the mathematical model of the problem in tables.

Keywords: transport problems (closed and open – with insufficiency or overstock), solution with Excel

1. Introduction

Transport problem is a specific type linear programming problem. Its essence is the following. It has to transport one type of product, which is supplied from several distributors, to several customers. It is preliminarily known the amount of the product keeping in store by every distributor and the respective quantities necessary for each customer. There are transport paths between each pair (distributor, customer). The aim of the problem is to minimize the total transport costs for transferring the products from each distributor to each customer. The definition of this problem is made very precise in [1], which is classical book for “Operation research”. It is being solved with simplex method, which guarantees an optimal solution of the problem. One program realization for solving the transport problem with Matlab is proposed in [2]. In this paper is suggested the solution of transport problem using Excel.

The paper is organized as it follows. The next Section 2 introduces definitions of closed and open transport problems. In Section 3 it considers the simplex method as a universal tool for solving linear programming problems (in particular transport problems). Four illustrative examples of transport problems (closed and open – with insufficiency or overstock – without and with blocked transportations) are solved using Excel. The paper finishes with conclusion remarks about advantages of Excel for solving this type of problems.

2. Problem Statement

Let’s have $n$ distributors $A_i$, $i = 1, 2, \ldots, n$ and $m$ customers $b_j$, $j = 1, 2, \ldots, m$. Each distributor keeps in store one type of product in respective quantities ($a_i$, $i = 1, 2, \ldots, n$) and each customer have a need of this product in respective quantity ($b_j$, $j = 1, 2, \ldots, m$). The cost for transportation the unit of the product from $i$-th distributor to $j$-th customer are $c_{ij}$, respectively. Then the objective function of the mathematical model of the transport problem (TP) has an effect of the total transportation costs and it has the following linear form:

$$f(x) = \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij}x_{ij} \rightarrow \min \quad (1a)$$

Constraints in the mathematical model of the classical closed transport problem are linear:

$$\sum_{j=1}^{m} x_{ij} = a_i, \quad i = 1, 2, \ldots, n \quad (1b)$$

$$\sum_{i=1}^{n} x_{ij} = b_j, \quad j = 1, 2, \ldots, m \quad (1c)$$

It has to be balanced between production kept in store and supplied production, i.e.

$$\sum_{i=1}^{n} a_i = \sum_{j=1}^{m} b_j \quad (1d)$$

Additional constraints in this model are:

$$0 \leq x_{ij} \leq d_{ij}, \quad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, m \quad (1e)$$

where $x_{ij}$ and $d_{ij}$ are the amount of production transported and the capacity of the channel from $i$-th distributor to $j$-th customer.

It is obvious ($1a$, $1b$, $1c$, $1d$, $1e$) that the mathematical model of closed TP is a linear model in the following form:
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(1a)

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It has to be balanced between production kept in store and supplied production, i.e.

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(1d)

Additional constraints in this model are:

$$0 \leq x_{ij} \leq d_{ij}, \quad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, m$$

(1e)

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It is obvious (1a, 1b, 1c, 1d, 1e) that the mathematical model of closed TP is a linear model in the following form:
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Meaning of each stage of algorithm is the following:

I. Initial stage, which consist of:

1. Setting up the initial simplex tableau (according to the identity matrix in constraints systems).

2. Determining the basis variables (according to the identity matrix in the constraints system) and setting up in simplex tableau.

II. Determine the basis solution as it follows. First $(2n+m)$ variables are so-called free variables and they set to 0. The basis variables are equal to the respective coefficients in the right part of the constraints system. Then it is calculated the objective function and it notes in the simplex tableau.

III. Verification of the basis solution for optimality:

— “yes” – The optimal solution is found.

— “no” – go to step IV.

IV. Change the basis solution.

V. Recalculate the simplex tableau.

4. Illustrative Examples of Transport Problems Solved with Excel

Module Solver in Excel is a powerful tool for solving linear optimization problems [4]. It uses simplex method for obtaining the final solution. The Solver Parameters dialogbox (Figure 2) needs to fill information about:

- Cells reserved for the decision variables.
- The cell where it calculates the value of the objective function.
- Cells where it is calculated the values for the left and the right sides of the constraints. It will also enter values for the right side of constraints and non-negativity constraints on the considered variables.

Next it has to set the type of optimization (max, min or going to exactly pre-defined value of the objective function) and the type of solving method. Each constraint is defined in the following dialog-box (Figure 3).

In this paper are solved 4 illustrative examples of TP – for closed and opened types, respectively.

4.1. Illustrative example 1 – Closed TP

Let’s have 3 distributors and 4 customers. A balance between production keeping in store and production supplied to the customers (1.d) is satisfied,
i.e. \( \sum_{j=1}^{n} a_{ij} = \sum_{j=1}^{m} b_{ij} = 720 \). The values of the coefficients \( c_{ij} \) in objective function are fulfilled in the first table. In the second one shows the solution of the TP with respect to the variables \( x_{ij} \). The values in the last row and the last column are total transported production from each distributor to each customer, respectively. So the solution of this problem is shown in Figure 4.

4.2. **Illustrative example 2 – Opened TP with insufficiency**

Let's have 3 distributors and 4 customers. In this example the supply is smaller than the consumption, i.e. equation (2b) is satisfied. Then it introduces additional fictive distributor with 0 values of \( x_{ij} \) in the row associated with this distributor. This fictive distributor returns the balance (in (2b)) between production kept in store and production supplied to the customers (1d).

The solution of this TP is shown in Figure 5.

4.3. **Illustrative example 3a – Opened TP with overstock**

Let's have 3 distributors and 4 customers. In this example the supply is bigger than the consumption, i.e. equation (5b) is satisfied. Then it introduces additional fictive customer with 0 values of \( x_{ij} \) in the column associated with this customer. This fictive customer returns the balance (in (3b)) between production kept in store and production supplied to the customers (1d).

The solution of this TP is shown in Figure 6.

4.4. **Illustrative example 3b – Opened TP with overstock and blocked transportations**

This example is the same as in example 3a with only difference to that some of transport connections between distributors and customers do not exist. So there is no paths between (Distributor 2 and Customer 1) and (Distributor 3 and Customer 1).

The solution of this TP is shown in Figure 7.

The values of the objective function change in solutions of the considered TP in examples 1, 2 and 3a, 3b. In closed TP the final value of objective function is 3740. This value increases to 4700 in opened TP with insufficiency because of the fact that the products in distributors are insufficient to satisfy the needs of the consumers and it introduces a fictive distributor. The opposite – in opened TP with overstock the total cost decreases because the needs of the consumers are smaller than the availabilities of...
the products in the distributors' storages and it introduces a fictive consumer. Then it transports only part of total product amount in storages which leads to reducing the total transportation cost to 3180. Example 3b is a particular case of Example 3a where some of the transportation paths are blocked and cannot be used for transportation. Therefore in this case the total cost decreases to 2990.

5. Conclusion

The proposed in this paper automatic solution of the transport problems with Excel spreadsheet gives to the users some advantages as significantly reducing the calculation time in comparison with respective solution getting by simplex method manually; good visualization and interpretation of the mathematical model of the considered problem, which is presented in Excel tables; user-friendly interface of optimization module of Excel, based on the classic simplex method and possibilities for sensitivity analysis of the solution varying with model's coefficients.

References


